9.4 Compare Linear, Exponential, and Quadratic Models

- Students will Compare Linear, Exponential, and Quadratic Models

Identifying from an equation:

Linear	Exponential	Quadratic
Has an x with no exponent. HOY $y = 5$ $y = 5x + 1$ $y = ½x$ $2x + 3y = 6$	Has an x as the exponent. $y = 3^x + 1$ $y = 5^{2x}$ $4^x + y = 13$	Has an x^2 in the equation; the highest power is 2. $y = 2x^2 + 3x – 5$ $y = x^2 + 9$ $x^2 + 4y = 7$

Examples:

- LINEAR, QUADRATIC or EXPONENTIAL?

a) $y = 6^x + 3$ — Exponential Growth

b) $y = 7x^2 + 5x - 2$ — Positive Quadratic

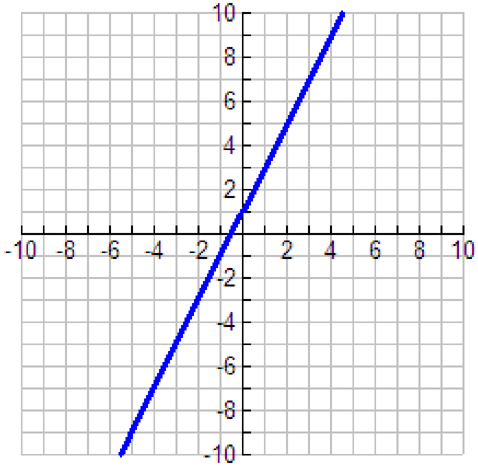
c) $9x + 3 = y$ — Increasing Linear

d) $4^{2x} = 8$ — Exponential Growth

Identifying from a graph:

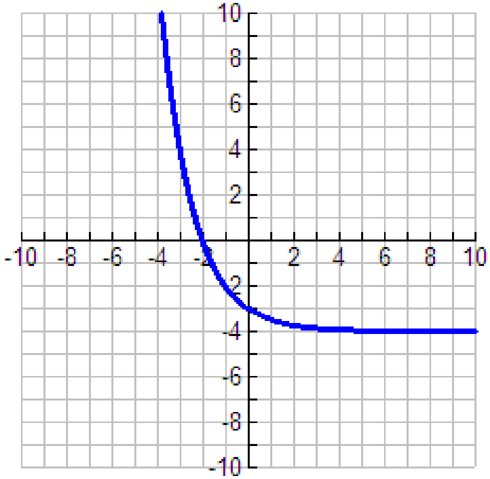
Linear

Makes a straight line



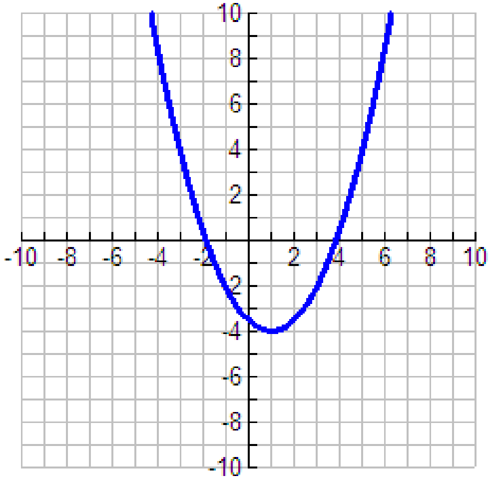
Exponential

Rises or falls quickly in one direction



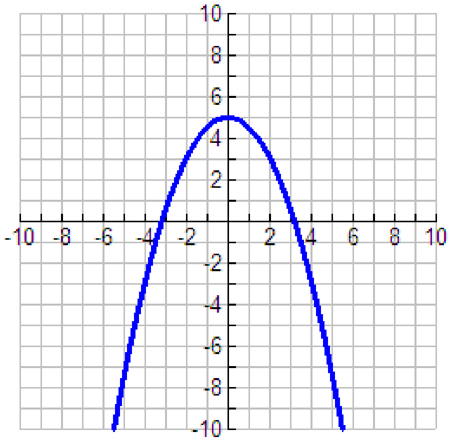
Quadratic

Makes a U or ∩ (parabola)



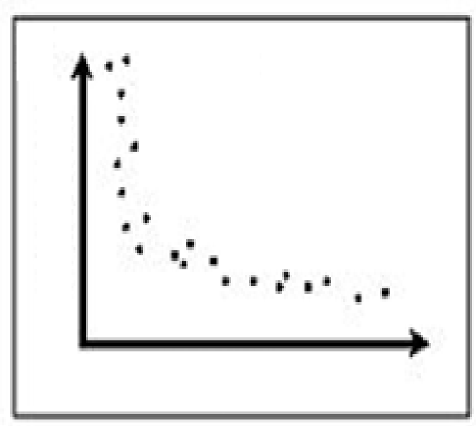
LINEAR, QUADRATIC or EXPONENTIAL?

a)



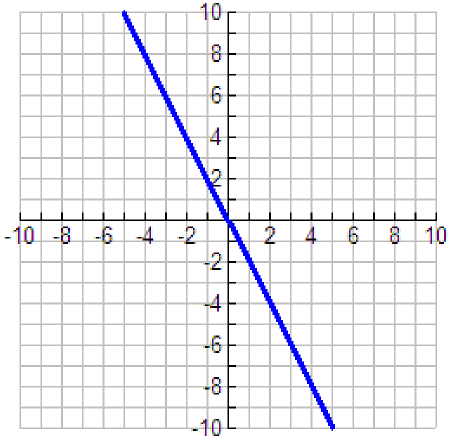
a) Negative Quadratic

b)



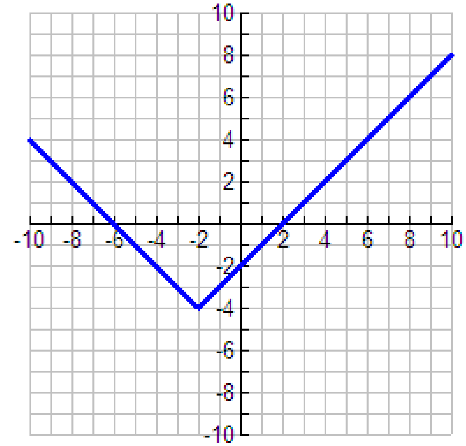
b) Exponential Decay

c)



c) Decreasing Linear

d)



d) Neither (Absolute Value)

Is the table linear, quadratic or exponential? All x values must have a common difference

Linear

- Never see the same y value twice.
- 1st difference is the same for the y values

Exponential

- y changes more quickly than x.
- Never see the same y value twice.
- Common ratio for the y values

Quadratic

- See same y more than once.
- 2nd difference is the same for the y values

Remember!

When the independent variable changes by a constant amount,

- linear functions have constant first differences.
- quadratic functions have constant second differences.
- exponential functions have a constant ratio.

Identify functions using differences or ratios

b.

x	-2	-1	0	1	2
y	-2	1	4	7	10

Differences: 3 3 3 3

ANSWER

The table of values represents a linear function.

EXAMPLE 2 **Identify functions using differences or ratios**

Use differences or ratios to tell whether the table of values represents a *linear function*, an *exponential function*, or a *quadratic function*.

a.

x	–2	–1	0	1	2
y	–6	–6	–4	0	6

First differences: 0 2 4 6

Second differences: 2 2 2

ANSWER

The table of values represents a quadratic function.

GUIDED PRACTICE for Examples 1 and 2

2. Tell whether the table of values represents a *linear function*, an *exponential function*, or a *quadratic function*.

x	– 2	– 1	0	1
y	0.08	0.4	2	10

ANSWER exponential function

Example 3: *Problem-Solving Application*

PROBLEM SOLVING

Use the data in the table to describe how the number of people changes. Then write a function that models the data. Use your function to predict the number of people who received the e-mail after one week.

E-mail forwarding	
Time (Days)	Number of People Who Received the E-mail
0	8
1	56
2	392
3	2744

3 Solve

Step 1 Describe the situation in words.

E-mail forwarding	
Time (Days)	**Number of People Who Received the E-mail**
0	8
1	56
2	392
3	2744

+ 1 + 1 + 1

× 7 × 7 × 7

This is an example of exponential growth.

Step 2 Write the function.

There is a constant ratio of 7. The data appear to be exponential.

$y = ab^x$ *Write the general form of an exponential function.*

$y = a(7)^x$ *Plug in the common ratio for b.*

$y = 8(7)^x$ *Plug in your initial (starting) amount for a.*

This is your model.

Step 3 Predict the e-mails after 1 week.

$y = 8(7)^x$ *Write the function.*

$= 8(7)^7$ *Substitute 7 for x (1 week = 7 days).*

$= 6{,}588{,}344$ *Use a calculator.*

There will be 6,588,344 e-mails after one week.

Check It Out! Example 3

Use the data in the table to describe how the oven temperature is changing. Then write a function that models the data. Use your function to predict the temperature after 1 hour.

Oven Temperature				
Time (min)	0	10	20	30
Temperature (°F)	375	325	275	225

3 Solve

Step 1 Describe the situation in words.

Oven Temperature

Time (min)	Temperature (° F)
0	375
10	325
20	275
30	225

+ 10   – 50
+ 10   – 50
+ 10   – 50

This is an example of a decreasing linear function.

Step 2 Write the function.

There is a constant reduction of 50° each 10 minutes. The data appear to be linear.

$y = mx + b$ *Write the general form of a linear function.*

$y = -5(x) + b$ *The slope m is –50 divided by 10.*

$y = -5(0) + b$ *Choose an x value from the table, such as 0.*

$y = 0 + 375$ *The starting point is b which is 375.*

$y = 375$

$y = -5x + 375$ *This is your model.*

Step 3 Predict the temperature after 1 hour.

$y = -5x + 375$ *Write the function.*

$= -5(60) + 375$ *Substitute 60 for x.*

$= 75^\circ$ F *Simplify.*

The temperature will be 75° F after 1 hour.