Honors Geometry
Lessons 2-2/2-3 Notes

1. **conditional statement**
   
   * an if-then statement
   * **hypothesis** ➔ the phrase after the word "if"
   * **conclusion** ➔ the phrase after the word "then"

**EXAMPLE 1**—Identifying the Hypothesis and the Conclusion
Identify the hypothesis and the conclusion of the following conditionals.

A. We will go swimming if the weather is nice. **If the weather is nice, then we will go swimming.**
   
   Hypothesis: the weather is nice
   
   Conclusion: we will go swimming

B. If I ride my bike to softball practice, then I will get there on time. **I will get there on time**
   
   Hypothesis: I ride my bike to softball practice
   
   Conclusion: I will get there on time
EXAMPLE 2—Rewriting in if-then form
Rewrite the conditional statement in if-then form.

A. Three points are coplanar if they lie on the same plane.
   If 3 points lie on the same plane, then they are coplanar.

B. Water freezes at temperatures below 32°F.
   If the temperature is below 32°F, then water freezes.

C. An even number is divisible by 2.
   If a number is even, then it is divisible by 2.

EXAMPLE 3—Writing a Counterexample
Write a counterexample to show that the following conditional statement is false.

If \( x^2 = 16 \) then \( x = 4 \).

1. Assume to be true
2. Find another conclusion that works.
   \( x = -4 \)

2. Converse
   * Switches the hypothesis and the conclusion

3. Inverse
   * The negation of the original conditional

4. Contrapositive
   * The negation of the converse (if original is true, contrapositive is true)
EXAMPLE 3—Writing a Converse, an Inverse, and a Contrapositive
Write the inverse, converse, and contrapositive of the statement.

A. If the sun is shining, then we are not watching TV.

INVERSE:
If the sun is not shining, then we are watching TV.

CONVERSE:
If we are not watching TV, then the sun is shining.

CONTRAPOSITIVE:
If we are watching TV, then the sun is not shining.

B. If my allowance increases, then I can save more money.

INVERSE:
If my allowance does not increase, then I can’t save more money.

CONVERSE:
If I can save more money, then my allowance increases.

CONTRAPOSITIVE:
If I can’t save more money, then my allowance didn’t increase.
5. **biconditional statement**

* A single true statement joining a true conditional and its true converse.

* Keeps original hypothesis & conclusion, replaces if-then with “if and only if”

**EXAMPLE 4—Rewriting a Biconditional Statement**

Rewrite the biconditional as a conditional and its converse.

**A.** An angle is a straight angle if and only if its measure is 180°.

**Conditional:**

If an angle is a straight angle, then its measure is 180°.

**Converse:**

If its measure is 180°, then an angle is a straight angle.

**B.** Two angles are supplementary if and only if the sum of their measures is 180°.

**Conditional:**

If 2 angles are supplementary then the sum of their measures is 180°.

**Converse:**

If the sum of their measures is 180°, then 2 angles are supplementary.
EXAMPLE 5—Writing a Biconditional Statement
Each of the following statements is true. Write the converse of each statement and decide whether the converse is true or false. If the converse is true, combine it with the original statement to form a true biconditional statement. If the converse is false, state a counterexample.

A. If $\sqrt{x} = 1$, then $x = 1$.
Converse: If $x = 1$ then $\sqrt{x} = 1$. TRUE.

Biconditional: $\sqrt{x} = 1$ if and only if $x = 1$.

B. If two angles are vertical angles, then they are congruent.
Converse: If two angles are congruent, then they are vertical angles. FALSE!

Counterexample: \[ \begin{array}{c}
\text{\angle A} \\
\text{\angle B}
\end{array} \quad \begin{array}{c}
\text{\angle C} \\
\text{\angle D}
\end{array} \]

EXAMPLE 6—Analyzing a Biconditional Statement
Consider the following statement: $x = 2$ if and only if $3x + 5x = 10x - 2x$. Is the statement true? Explain why or why not.

Conditional: If $x = 2$ then $3x + 5x = 10x - 2x$. TRUE

Converse: If $3x + 5x = 10x - 2x$ then $x = 2$. FALSE
6. **symbolic notation**

- $p \rightarrow q$ \textit{conditional} \text{\textquotedblright} “if $p$ then $q$”
- $q \rightarrow p$ \textit{converse} \text{\textquotedblright} “if $q$ then $p$”
- $\sim p \rightarrow \sim q$ \textit{inverse} \text{\textquotedblright} “if not $p$ then not $q$”
- $\sim q \rightarrow \sim p$ \textit{contrapositive}
- $p \leftrightarrow q$ \textit{biconditional}

**EXAMPLE 7—Using Symbolic Notation**

Let $p$ be “the value of $x$ is 7” and let $q$ be “$x$ is less than 10.”

A. Write $p \rightarrow q$ in words.

\begin{itemize}
\item If the value of $x$ is 7 then $x$ is less than 10. \textbf{TRUE}
\end{itemize}

B. Write $q \rightarrow p$ in words.

\begin{itemize}
\item If $x$ is less than 10, then the value of $x$ is 7, \textbf{FALSE}
\end{itemize}

C. Decide whether the \underline{biconditional} statement $p \leftrightarrow q$ is true. If so, write the \underline{biconditional}.

\begin{itemize}
\item $p \leftrightarrow q$ is not true because the converse is not true.
\end{itemize}
EXAMPLE 8—Identifying Good Definitions

Is the following statement a good definition? Explain why or why not.

A square is a figure with four right angles.

**And:** If a figure is a square, then it has 4 right angles. **T**

**Converse:** If a figure has 4 right angles, then it is a square. **F**

This is not a good definition b/c it cannot be written as a biconditional.